Scale-free and small-world networks in geographical research: A critical examination

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SCALE-FREE AND SMALL-WORLD NETWORKS IN GEOGRAPHICAL RESEARCH: A CRITICAL EXAMINATION[1]

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ABSTRACT
A rapid surge of interest for networks in the late 1990s throughout natural and social sciences has witnessed the emergence and the diffusion of new concepts and measures. In this paper, we wish to examine how two recent models of networks (i.e. scale-free and small-world) have been integrated in the works of geographers, what have been the benefits, and whether such concepts are likely to increase their influence in further works on networks. First, we propose a critical review of the 'scale-free' and 'small-world' concepts, notably based on a review of the physics literature. The second section examines the spatial dimension in networks studies, and the third one evaluates how geographers have used these measures and concepts in their works. In conclusion, we question the benefits of these two models of networks to geography compared with other approaches such as the ones developed in sociology.

KEY-WORDS
Scale-free network, small-world network, geography, regional science, social network analysis

1. INTRODUCTION

A rapid surge of interest for networks in the late 1990s throughout natural and social sciences has witnessed the emergence and diffusion of new concepts and measures. However, little has been done evaluating the impact and outcome of the latter on geography, with reference to the recent reviews by Borgatti *et al.* (2009) and Crossley (2005, 2008) about sociology. In this paper, we wish to examine how two recent models of networks (i.e. scale-free and small-world) have been integrated in the works of geographers, what have been the benefits, and whether such concepts are likely to increase their influence in further works on networks. In a first attempt to evaluate the benefits of these approaches in geography, Rozenblat and Mélançon (2007) noticed that "this type of empirical approach combining a conceptual approach of 'small world theory' and dedicated tools has not been developed in geography": is the situation different four years later?

Since the quantitative revolution of the 1960s, the status of network analysis in geography has remained rather simple. This tool is mainly used by transport geographers focusing primarily on graph theory with applications to planar and technical networks (e.g., roads and railways, see Kansky, 1963; Chorley and Haggett, 1969; Dupuy, 1988; Mathis, 2003). Relations with Social Network Analysis (SNA) remain limited (Freeman, 2004:85) although sociologists developed their tools from the 1920s while focusing exclusively on non-planar graphs (Wasserman and Faust, 1994). Network analysis in geography has thus relatively stagnated notwithstanding advances in Geographical Information Systems (GIS), while regional scientists continued developing their own models on spatial analysis and flow optimisation (Waters, 2006).

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The sudden interest by physicists in network analysis in the late 1990s principally provided models of networks based on two main dimensions: the small-world network, based on average distance path and density of neighbourhoods (Watts and Strogatz, 1998; Watts, 2003), and the scale-free network based on the hierarchy of hubs (Barabási and Albert, 1999). The first concept originates from the famous experience of Stanley Milgram (1967) which had very limited diffusion in geography except from a paper by Stoneham (1977). While such concepts have quickly spread across various disciplines and scientific fields (Newman *et al.*, 2006), their integration in geography still appears rather limited to a few studies in transport, urban and economic geography. Conversely, physicists have increasingly integrated the spatial dimension in their works (see the extensive review of Barthélemy, 2010).

The structure of the paper is as follows. First, we propose a critical review of the 'scale-free' and 'small-world' concepts based on a review of the physics literature. The second section examines the space dimension in networks studies using these two models, and the third one studies how geographers have used these measures and concepts in their works. In conclusion, we question the benefits of these measures to geography compared with other approaches such as the ones developed in SNA, which have been far less integrated by geographers despite their *a priori* relevance at *meso* (regional) levels of analysis (Grabher, 2006). This research also provides floor for further discussing how geography integrates (or not) scientific innovations.

2. SCALE-FREE AND SMALL-WORLDS NETWORKS

Most researches in graph theory and network analysis in general have long been focused primarily on regular and random networks. The concepts of 'small-world network' (hereafter SWN) and 'scale-free network' (SFN) were first proposed by the respective works of Watts and Strogatz (1998) and Barabási and Albert (1999). Their goal was to define models of network organisation differing from regular and random networks (Erdös and Rényi, 1959). Random networks are characterised by a normal degree distribution, whereas SFNs contain few large degree nodes and a majority of small degree nodes, resulting in a strong hierarchical structure. SWNs exhibit a small average path length between pairs of nodes and a high local clustering coefficient (also called transitivity: probability for nodes that their adjacent neighbours are linked). SWNs and SFNs have in common several features (Newman *et al.*, 2006):

- They are more efficient in terms of ease of circulation within the network, which can be measured by the average shortest path length, as those networks allow limiting the number of stops between two nodes on average;

- They include many hubs (bridge nodes) and isthmuses (crucial links) between densely and tightly connected communities or 'clusters', based on the idea of cliques.

But such networks also have some important differences:

- SFNs are less 'clustered' than SWNs due to the stronger influence of large degree nodes, which lowers the clustering coefficient (cf. hub-and-spoke configurations in air transport). SFNs seem more efficient as the presence of hubs provides optimal circulation and less friction;

- SWNs are denser than SFNs because removing few hubs would result in the identification of communities. Yet, large degree nodes tend to form cliques in SFNs and this can be measured by the rich-club coefficient.

Usually, the structure of SFNs is described by plotting node frequency over degree distribution in a log-log diagram. The slope (exponent) of the power-law line gives an indication whether the network is scale-free, i.e. when values of the exponent are over 1 or even 2. Another way testing the existence of scale-free properties is dynamic. As SFNs are evolving by preferential attachment, when new nodes are added to the network, they primarily connect the already large nodes, which function as magnets towards new entrants, thereby reinforcing the hierarchical structure. Such dynamical properties inherent to SWNs and SFNs were in fact already expressed in a number of related concepts and growth models, as underlined by the recent review provided by Zaidi (2011) on complex networks:

- Scale-free: the power-law structure and the preferential attachment process were already described by the Yule (1925) process, by the Gibrat (1931) law (growth is proportional to size), by Jackson (1935) with the 'rich get richer' idea, by Zipf (1949) on the rank-size rule, by Price (1965, 1976) about the cumulative advantage process observed in citation networks, and by Merton (1968) about inequality in credit attribution among researchers;

- Small-world: the works of Milgram (1967) as well as Travers and Milgram (1969) were in fact the first to demonstrate the topological proximity among distant individuals and to label it as „small-world phenomenon‟. Earlier, Simmel and Wolff (1950) first proposed the concept of triad to depict mutual acquaintances in a social network, and how they are likely to evolve.

In fact, the merit of recent works by Watts and Strogatz (1998) and Barabási and Albert (1999) has been to reincorporate such ideas into clearly defined models of network structure and evolution, together with associated measures and methods directly usable for empirical research. Several measures have been proposed to highlight the properties of networks and the following list does not pretend to be exhaustive:

- Nodal hierarchy: fitness of degree distribution with power-law trend, that is similar to the rank-size rule proposed by Zipf (1949) and widely applied in urban studies for decades;

- Transitivity: probability for nodes to have their adjacent neighbours connected with each other, measured by the 'global clustering coefficient'. Although it is not explicitly addressed, this measure actually refers to the fraction of closed triplets proposed by sociologists decades ago (transitivity). This indicator is also measured at the node level by the 'local clustering coefficient';

- Efficiency: average shortest path length is the average number of links needed to connect all node pairs in the graph;

- Assortativity: refers to the global measure of 'degree correlation' depicting the level of homogeneity of the network's structure. It is the result of the Pearson correlation between degree scores of nodes for all links;

- Density, connectivity: the „rich-club coefficient‟ proposed by physicists is a derivative of the γ index proposed by Kansky (1963), also called density by social network analysis (SNA) researchers , applied to links between nodes over a certain degree threshold.

In the end, physicists have mostly relied on existing measures, but they have also modified and improved them in order to take the weights into account, which has long remained a drawback of graph theory (Opsalh and Panzarasa, 2009). Weighting, for instance, the clustering coefficient and the rich-club coefficient provide very useful answers to the question whether larger nodes are more strongly interconnected with each other than with smaller nodes, i.e. whether hubs form cliques in the graph. If weight is taken into account, directionality is barely considered by physicists, even when links are obviously directed (cf. Barábasi's studies of the Internet structure), which is less the case in SNA where main methods are suited for directed graphs (blockmodeling, equivalence, Siena model, etc.).

3. INTEGRATING SPACE IN NETWORK RESEARCH

A large number of research papers seeking small-world and scale-free properties in networks of all kinds has been produced in the last two decades at exponential pace, thus making it difficult reviewing the field exhaustively. Given the similarity of methods and measures from one case study to the other, we review a number of works principally focusing on the spatial dimension of SFNs and SWNs.

Natural scientists have themselves criticized mainstream research on networks due to the non-inclusion of the spatial, social or economic dimension in general measures and models (Watts, 2003). The material and geographical embedding of some networks appears only implicitly in the small-world model, while it is absent of the scale-free model[2]. In addition, Watts (1999) found that preferential attachment often occurs over shorter distances. A distinct category of 'spatial networks'[3] thus emerged in an explicit way in the physics literature, which actually applies to a majority of real-world networks in contrast with theoretical models of networks (Gastner and Newman, 2004). Yet, many works on spatial networks do not specifically measure their spatial dimension, as they only look at the topological dimension as in the case of maritime networks (Hu and Zhu, 2009) and commuter flows (De Montis *et al.*, 2010) where distance parameters are not included. Spatial networks are defined by physicists (Barthélemy, 2010) through some fundamental properties:

- Physical embedding: nodes and/or links are grounded in a physical (Euclidian) space, which in turn constrains the multiplication of links and orientates the layout of the network, with the crucial importance of borders;

- Interaction range: distance metrics (and related costs) play a central role in the emergence, distribution, and weight of links, since spatial proximity is one dominant factor favouring short-range versus long-range interaction.

Several scholars have thus explored the influence of spatial structure on network topology in static and dynamics ways (Waters, 2006). Among the earliest attempts to validate this idea, Barthélemy (2003 and 2010) showed that spatial networks in general exhibit higher clustering coefficients than non-spatial networks due to the importance of proximity in node connectivity. Further results were provided by Barrat *et al.* (2005) based on the case of air transport highlighting several other properties of spatial networks such as: fewer global hubs and more regional hubs, higher disassortativity as the network grows, higher correlation between degree and betweenness, increased influence of the barycentre on betweenness values, and increased cliquishness. However, the fundamental difference between planar and non-planar networks is not always considered by physicists. For instance, planar spatial networks are more physically constrained and thus are more assortative, with a higher probability to contain a giant component (i.e. connected subgraph including a majority of the nodes), while non-planar spatial networks are more likely to exhibit scale-free properties (Bullock et *al.*, 2010).

Some recent interest toward planar graphs can be seen among physicists, and the class of 'Apollonian networks', being simultaneously planar, scale-free and small-world, illustrates it (Andrade *et al.*, 2005). It seems too soon however to check the empirical relevance of this new network model.

Methods used for testing the influence of spatial embedding range from the inclusion of simple distance parameters to the simulation and modelling of complex networks embedded in two-dimensional space (for an early review see Boccaletti *et al.*, 2006, pp. 205-212), based on the hypothesis that connectivity is a function of distance (Barnett *et al.*, 2007). Notably, Crucitti *et al.* (2006) take into account physical distance in their calculation of node centrality, arguing that their

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[2] Barabási (2002) recognized that cities connecting thousands of highways simply do not exist, thereby recognizing implicitly physical constraints to network growth.

[3] Spatial networks are also coined geographical networks, technological networks, infrastructure networks, *ad-hoc* networks, or physical networks in the literature.

results may be more useful to urban planning and design. In the same vein, Cardillo *et al.* (2010) include metric distance in their analysis of urban streets, notably comparing observed and optimal efficiency. Other works adopted a node redundancy approach to study the influence of spatial structure on cascading failures (Huang *et al*., 2006), arguing that stronger geographical constraints foster the 'reservoir effect' of hubs (i.e. redistribution of traffic from smaller nodes to larger nodes situated in close proximity), while such spatial networks face higher risks of becoming disconnected due to their higher density. When it comes to the simultaneous analysis of several spatial networks, Parshani *et al.* (2010) notably demonstrated that location matters in the inter-similarity of networks, based on ports and airports' geographical coordinates.

4. SCALE-FREE AND SMALL-WORLD NETWORKS IN GEOGRAPHY AND REGIONAL SCIENCE

While the quantitative revolution of the 1960s favoured a rapid diffusion of graph theoretical concepts and methods, network analysis in geography stagnated during the following decades. The wider paradigm shift from structural to behavioural approaches is seen by Waters (2006) as a main cause for the declining interest in spatial analysis as a whole, where network analysis 'remained somewhat of a backwater' despite improvements provided by Geographical Information Systems (GIS) since the 1980s. Since the early 1990s, transport geographers observed the emergence of hub-and-spokes networks in various industries (e.g. airlines) without any reference to SFNs. In such context, geographers remained isolated from research on networks in other disciplines, relying on dated measures of network structure most of them coming from classic graph theory. This may also be explained by the absence of collaborations with sociology where network analysis has been at centre stage since the 1920s. Social geographers have often given privilege to qualitative methods rather than quantitative, since social geography emerged in opposition to the quantitative revolution.

Such picture has changed in the 1990s when geographers started to represent and analyse non-planar networks, thereby stepping out of 'classic' graph theory and thus needing more advanced tools to represent and analyse such networks. Studies of the European urban system progressively integrated a network dimension with the works of Cattan (1995) on airlines, and Rozenblat and Pumain (1993) on multinational firms. Yet, these works innovate both by the methods used for analysing geographical networks and by the paths they opened towards new ways considering systems of cities. Anyway, according to Gorman *et al.* (2004), few scholars have improved existing connectivity indices, although such progress remained hindered by a lack of knowledge about the history and development of network analysis in geography and in other disciplines (Waters, 2006). Although Gorman and Kulkarni (2004) consider Batty (2001) as the main propeller of complex network research within urban geography and economics, the latter is not always cited in the pioneering attempts by regional scientists to integrate a network perspective in their works (Capello and Nijkamp, 1995). Instead, regional scientists directly refer to physicians, as in the case of Jiang and Claramunt's work on urban streets as SWNs (2004).

A review of recent applications of scale-free and small-world networks by geographers and regional scientists provides a somewhat balanced picture of the varied outcomes. Several works done by geographers remain direct applications of complex network measures on various datasets, as in the case of airline networks, notwithstanding some innovation through the refinement of clustering methods (Amiel *et al.*, 2005), the comparison of centrality measures with local socio-economic data (Wang *et al.*, 2011), or the combination with maritime networks (Ducruet *et al.*, 2011). Distance or territorial aspects themselves are therefore barely introduced by geographers in their study of networks. It was found, however, that seaports with larger degree connect over longer kilometric distances on average based on worldwide maritime links (Ducruet and Zaidi, 2011). When studying the Indianapolis road network, Gleyze (2007) proposed to distinguish among networks effects and spatial effects in the measurement of betweenness centrality and eccentricity. Innovating thus takes place in the discipline without modifying or even discussing the benefits and limits of such methods and models. Other works, however, clearly use complex networks as a complement to their own existing methods, such as Patuelli *et al.* (2007) combining complex networks and Spatial Interaction

Models (SIM), Gorman and Kulkarni (2004) referring to SNA (e.g. structural equivalence) and complex methods in their analysis of Internet backbone networks in the US. In the same vein, Ducruet *et al.* (2011) apply successively three sets of methods on the combined air-sea global network: scale-free dimension of the network on different levels of node aggregation, Multiple Regression Quadratic Assignment Procedure (QAP) to reveal the correlation between network topologies, and the more classic 'nodal regions' algorithm[4] for highlighting relevant sub-trees in the network. For Vinciguerra *et al.* (2010), the goal is to better understand the evolution of the European Internet backbone network through the testing of the Barabási-Albert model of preferential attachment. Other works use complex networks as a tool to perform their analysis, such as Rozenblat (2010) works on the location logics of multinational firms in a multi-level perspective within and among cities, or Comin (2009) using available centrality measures (degree, closeness, betweenness) to depict the situation of European cities in partnership networks and their dynamics. The diachronic dimension is indeed much taken into account by geographers when it comes to actual data rather than simulation models.

5. CONCLUSION

SFNs and SWNs became quite quickly a popular hobby for physicists, and some included spatial issues in their research. Geographers remained mostly apart from this trend, despite some recent exceptions. While cross-disciplinary interactions between geography and other fields remained higher with physics (and computer sciences) than with sociology, most geographers kept on considering networks as planar and technical graphs.

There may be various causes to this state of affairs. On the one hand, the shift away from structural approaches in the 1970s can be seen as one of them. Yet, this cannot explain why quantitative geographers have not been faster in adopting SFNs and SWNs, as they did in the 1960s when integrating graph theory. On the other hand, one likely reason is the limited innovation brought by complex networks research to geography, but again, no geographer has clearly expressed such critique in a formal review of the field as it was done by the aforementioned sociologists.

Many tools exist nowadays (e.g. GIS, R packages) which can provide a valuable help to test measures and methods on networks. Geographers focusing on migration, economy, political geography, without mentioning transport geography, could surely examine further the relevance of these innovations. Questioning their relevance cannot be done without testing these measures. In addition, geographers have the opportunity to improve the integration of space (and time) in network research, which has many concrete applications and is being adopted by decision-makers as a relevant approach to their problems.

The works of physicists indeed still has some weaknesses despite their plethoric (and sometimes redundant) production: absence of critical discussion on data quality and relevance, limited knowledge on the specific study field, and shortage of results' interpretation and implications. Such drawbacks, combined with the trend to re-discover well-known measures by giving them a new name (the most obvious being the transitivity renamed global clustering coefficient), still cannot fully erode the potential benefits of SFN and SWN models. Further research in geography may insist on the necessity to analyze networks as elements of wider territorial structures. Can we identify socio-economic invariants in the hierarchy of places and in the emergence of dense communities in the network? Are there determinants of network evolution beyond the sole role of costs and Euclidian distances?

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[4] Also coined simple linkage analysis, or dominant flow analysis in the literature.

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